Standard -12| chapter-3| Theory of equations| youtube video class (presented by SHANKAR G)

Note: One touch the questions and to get the answers

Chapter 3 Theory of Equations

Example 3.1

If α and β are the roots of the quadratic equation $17x^2 + 43x - 73 = 0$, construct a quadratic equation whose roots are $\alpha + 2$ and $\beta + 2$.

Example 3.2

If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.

Example 3.3

If α, β, and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients.

Example 3.4

Find the sum of the squares of the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$, $a \neq 0$

Example 3.5

Find the condition that the roots of cubic equation $x^3 + ax^2 + bx + c = 0$ are in the ratio $p : q : r$.

Example 3.6

Form the equation whose roots are the squares of the roots of the cubic equation $x^3 + ax^2 + bx + c = 0$.

Example 3.7

If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.

EXERCISE 3.1

1. If the sides of a cubic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.

2. Construct a cubic equation with roots
   (i) $1, 2$, and 3 (ii) $1, 1$, and -2 (iii) $2, \frac{1}{2}$ and 1.

3. If α, β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, form a cubic equation whose roots are
   (i) 2α, 2β, 2γ (ii) $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ (iii) $-\alpha, -\beta, -\gamma$

4. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.

5. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$.

6. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio $3:2$.

7. If α, β, and γ are the roots of the polynomial equation $ax^3 + bx^2 + cx + d = 0$, find the value of $\sum \frac{\alpha}{\beta\gamma}$ in terms of the coefficients.

8. If α, β, γ, and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$, find a quadratic equation with integer coefficients whose roots are $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$.

9. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.

10. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$.

Example 3.8

Find the monic polynomial equation of minimum degree with real coefficients having $2 - \sqrt{3}i$ as a root.

Example 3.9

Find a polynomial equation of minimum degree with rational coefficients, having $2 - \sqrt{3}$ as a root.

Example 3.10

Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

Example 3.11

Show that the equation $2x^2 - 6x + 7 = 0$ cannot be satisfied by any real values of x.

Example 3.12

If $x^2 + 2(k+2)x + 9k = 0$ has equal roots, find k.

Example 3.13

Show that, if p, q, r are rational, the roots of the equation $x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$ are rational.

Example 3.14

Prove that a line cannot intersect a circle at more than two points.

EXERCISE 3.2

1. If k is real, discuss the nature of the roots of the polynomial equation $2x^2 + kx + k = 0$, in terms of k.

2. Find a polynomial equation of minimum degree with rational coefficients, having $2+\sqrt{3}i$ as a root.

4. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root.

5. Prove that a straight line and parabola cannot intersect at more than two points.

Example 3.15

If $2+i$ and $3-\sqrt{2}$ are roots of the equation

$$x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0,$$

find all roots.

Example 3.16

Solve the equation $x^4 - 9x^2 + 20 = 0$.

Example 3.17

Solve the equation $x^3 - 3x^2 - 33x + 35 = 0$.

Example 3.18

Solve the equation $2x^3+11x^2-9x-18=0$.

Example 3.19

Obtain the condition that the roots of $x^3+px^2+qx+r=0$ are in A.P.

Example 3.20

Find the condition that the roots of $ax^3+bx^2+cx+d=0$ are in geometric progression. Assume $a,b,c,d\neq 0$

EXERCISE 3.3

1. Solve the cubic equation : $2x^3-x^2-18x+9=0$ if sum of two of its roots vanishes.

2. Solve the equation $9x^3-36x^2+44x-16=0$ if the roots form an arithmetic progression.

3. Solve the equation $3x^3-26x^2+52x-24=0$ if its roots form a geometric progression.

4. Determine k and solve the equation $2x^3-6x^2+3x+k=0$ if one of its roots is twice the sum of the other two roots.

5. Find all zeros of the polynomial $x^6-3x^5-5x^4+22x^3-39x^2-39x+135$, if it is known that $1+2i$ and $\sqrt{3}$ are two of its zeros.

Answer: Refer example 3.15

6. Solve the cubic equations : (i) $2x^3-9x^2+10x=3$, (ii) $8x^3-2x^2-7x+3=0$.

Answer: Refer example 3.17 & 3.18

7. Solve the equation : $x^4-14x^2+45=0$.

Answer: Refer example 3.16

Example 3.23

Solve the equation

$$(x-2)(x-7)(x-3)(x+2)+19=0.$$

EXERCISE 3.4

1. Solve : (i) $(x-5)(x-7)(x+6)(x+4)=504$ (ii) $(x-4)(x-7)(x-2)(x+1)=16$
2. Solve : $(2x-1)(x+3)(x-2)(2x+3)+20=0$

Answer: Refer example 3.23